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TRANSIENT MOTION OF AN ELASTIC SHELL OF REVOLUTION IN AN ACOUST--ETC(U)
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Transient Motion of an Elastic Shell of of Revolution in an Acoustic Medium [1]

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10 B. S. BERGER [2]

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Introduction

The form of the hulls of many undersea craft is approximated by a closed surface of revolution. This property, together with the numerical solutions available for elastic shells of revolution, has motivated the study of these figures in connection with the acoustic radiation problem. See [1-3] [3]. Solutions have been given which involve various simplifying assumptions regarding the nature of the radiated acoustic field. [4-6]. Ref. [4] contains a summary of some of the recent work in this area.

In the following, the scaled acoustic field equations are expressed in circular cylindrical coordinates and the circumferential coordinate suppressed through a Fourier series expansion. The region external to shell's generating a curve is mapped conformally onto the region external to the unit circle. This is tantemount to the construction of an orthogonal coordinate system which has the shell's generating curve as one of its coordinate lines. The numerical conformal mapping technique utilized is applicable to infinite regions bounded internally. The region external to the unit circle is mapped into the region internal to a rectangle. The equation of motion of the acoustic fluid together with the shell fluid boundary equations in the transformed coordinate system is expressed in finite difference form.

The normal shell displacement is introduced through a spectral representation, as a series of the in vacuo shell modes. Integrals occurring in forcing terms of the expansion which contain the unknown surface acoustic pressure

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3 Numbers in brackets designate references at the end of the paper.

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Abstract

A numerical solution for the *transient* vibration of an arbitrary shell of revolution, surrounded by an acoustic medium, has been formulated in terms of the conformally mapped acoustic equations, fluid shell boundary conditions and spectral representation of the shell displacements. The technique readily includes the boundary condition at infinity, internal shell structure and is exact to within those approximations implicit in the finite difference method.

where $T = ct/a$, $P = p/\rho_f c^2$ and ρ_f = fluid density. In (9), the angular coordinate, x_2, may be suppressed through a Fourier expansion of P

$$P(x_1,x_2,x_3,T) = \sum_{n=0}^{\infty} (P_{nc} \cos nx_2 + P_{ns} \sin nx_2) \tag{10}$$

For notational simplicity, P_n will be taken, in the following, to represent either P_{nc} or P_{ns}. For the exterior problem $x_1 \geq 1$. However, for a closed shell of revolution there exist points on the shell's surface for which $X_1 = 0$. The corresponding singularities in (9) may be eliminated through the change of dependent variable

$$P_n = F_n / X_1^2 (1,x_3) \tag{11}$$

The region external to the unit circle $x_1 = 1$ may be mapped into the interior of a rectangle through the inversion

$$y = 1/x_1 \tag{12}$$

See [1,2,3]. Substituting (10), (11) and (12) into (9) and taking the Laplace transform with respect to T, assuming zero initial conditions gives

$$\frac{\partial^2 Fn}{\partial y^2} + a_1' \frac{\partial Fn}{\partial y} + a_2' \frac{\partial Fn}{\partial x_3} + a_3' \frac{\partial^2 Fn}{\partial x_3^2} + a_4' F_n = 0 \tag{13}$$

where $a_1' = 1/y - (\partial X_1/\partial x_1)/(y^2 X_1)$, $a_2' = (\partial X_1/\partial x_3)/(X_1 y^2) - 4(\partial X_1(1,x_3)/\partial x_3)/(y^2 X_1(1,x_3))$; $a_3' = 1/y^2$, $a_4' = (6(\partial X_1(1,x_3)/\partial x_3)^2/X_1(1,x_3) - 2(\partial^2 X_1(1,x_3)/\partial^2 x_3^2))/(y^2 X_1(1,x_3)) - 2(\partial X_1/\partial x_3)(\partial X_1(1,x_3)/\partial x_3)/(y^2 X_1 X_1(1,x_3)) - (g_{11}/y^4)(s^2/a^2 + n^2/X_1^2)$ and s is the Laplace transform variable.

An effective method for computation of the coefficients a_n occurring in the mapping function (3) is given in [10]. This technique has been applied to the solution of the cylindrical problem in [7].

Eq. (13) may be expressed in finite difference form by replacing the spatial derivatives with the following difference expressions:

$F,_{x|x=x_2} \simeq C_n(x_2)F_n$, n=1,3, $C_1 = 1/\beta$, $C_2 = (\alpha^2-1)/\beta$, $C_3 = -\alpha^2/\beta$, $\beta = \alpha(\alpha+1)h$, $h = x_2 - x_1$ and $\alpha h = x_3 - x_2$. For points not adjacent to y = 0, y = 1, $x_3 = \pm\pi/2$ the second derivative was approximated by $F,_{xx|x=x_2} \simeq A_n(x_2)F_n$, n=0,4 where the mesh points are the set $\{x_o,x_1,x_2,x_3,x_4\}$, $A'_n = \Sigma\, x_m$, m=0,4, $m \neq n$, $B_n = (A'^2_n - D_n)/2$, $C_n = \Sigma(x_n - x_m)$, m = 0,4, $m \neq n$, $D_n = x_m x_m$, m = 0,4, $m \neq n$, and $A_n(x_2) = 12\, x_2^2 - 6A'_n\, x_2 + 2B_n)/C_n$. For points adjacent to y = 0, y = 1, $x_3 = \pm\pi/2$ the second derivative was approximated by $F,_{xx|x=x_1} = B_n(x_1)F_n$ n = 0,3 where $A_n = \Sigma\, x_m$, m = 0,3, $m \neq n$, $C_n = \Sigma(x_n - x_m)$, m = 0,3, $m \neq n$, $B_n(x_1) = (6x_1 - 2A_n)/C_n$ and the mesh points consist of the set $\{x_o,x_1,x_2,x_3\}$. At a general nodal point $y = y_i$, $x_3 = x_{3k}$ the finite difference form of (13) is then

$$\sum_{r=-2}^{+2} (D_{i+r,k}\, F_{i+r,k} + D'_{i,k+r}\, F_{i,k+r}) = 0 \qquad (14)$$

where the coefficients of F in (14) are found by substituting the above difference expressions into (13) and the subscript n has been omitted in the symbol F_n for the sake of clarity.

Boundary Conditions

At the shell fluid interface, the normal fluid velocity must equal the normal shell velocity. This implies that

$$a(\partial P_n/\partial \bar{n} + \partial P_{nI}/\partial \bar{n}) = -\partial^2 W_n/\partial T^2 \qquad (15)$$

where P_n, P_{nI} and W_n are the coefficients of the Fourier expansions, in terms of x_2, of the scaled reflected and radiated acoustic pressure, the scaled known incident acoustic pressure and the scaled normal shell displacement respectively. $\bar{n}$ is an exterior unit vector which is normal to the shell's surface in the unscaled physical coordinates x_i. It may be shown, [8,9] that

$$\frac{\partial P}{\partial \bar{n}} = n^k \frac{\partial P}{\partial x_k} \tag{16}$$

where n^k are the contravarient components of the unit vector $\bar{n}$ in the x_i coordinate system. The tensor transformation law implies that

$$n^k = \frac{\partial x_k}{\partial \bar{x}_i} \bar{n}^i \tag{17}$$

Since $\bar{n}$ is normal to a surface of revolution and $\bar{x}_i$ is cylindrical it follows that $\bar{n}^2 = 0$. This and the fact that x_1 and x_2 are only functions of $\bar{x}_1$ and $\bar{x}_3$, together with (17), implies that $n^2 = 0$. For covarient components (17) becomes

$$n_k = \frac{\partial \bar{x}_i}{\partial x_k} \bar{n}_i \tag{18}$$

For orthogonal coordinates $\bar{n}^i = 0$ implies that $\bar{n}_i = 0$ and therefore $\bar{n}_2 = 0$. Summing (18) for k = 3 gives $n_3 = (\partial\bar{x}_1/\partial x_3)\bar{n}_1 + (\partial\bar{x}_3/\partial x_3)\bar{n}_3$. However $\bar{n} = \bar{\nabla}f/|\bar{\nabla}f| = ((\partial f/\partial\bar{x}_1)\bar{g}_1 + (\partial f/\partial\bar{x}_3)\bar{g}_3)/\bar{\nabla}f = \bar{n}'\bar{g}_1 + \bar{n}^3\bar{g}_3$ and $\bar{n}' = \bar{n}_1$, $\bar{n}^3 = \bar{n}_3$ since $\bar{n}^i = \bar{g}^{ij}\bar{n}_j$, $\bar{g}^{ij} = 0$, $i \neq j$, $\bar{g}^{11} = \bar{g}^{33} = 1$. Here $f(\bar{x}_1,\bar{x}_3) = c$ is the equation of the generating curve. Then $n_3 = ((\partial\bar{x}_1/\partial x_3)(\partial f/\partial\bar{x}_1) + (\partial\bar{x}_3/\partial x_3)(\partial f/\partial\bar{x}_3))/|\bar{\nabla}f| = (\partial f/\partial x_3)/|\bar{\nabla}f| = 0$ since $f(1,x_3) = c$ is the equation of the generating curve in the x_i coordinate system. Then $n_3 = 0$ and therefore $n^3 = 0$. Since n is a unit vector it follows from the definition of the scalar product and the definition of the non-dimensional coordinates x_i that $n^i n^k g_{ik} = 1/a^2$. Since $n^2 = 0$ and $n^3 = 0$, it follows that $(n^1)^2 g_{11} = 1$ or $n^1 = 1/(a\sqrt{g_{11}})$. Substituting into (16) gives

$$\frac{\partial P}{\partial \bar{n}} = \frac{1}{a\sqrt{g_{11}}} \frac{\partial P}{\partial x_1} \tag{19}$$

The boundary condition (15) is then

$$\frac{-1}{\sqrt{g_{11}}\, X_1^2(1,x_3)} \frac{\partial F_n}{\partial y} + a \frac{\partial P_{nI}}{\partial n} = - \frac{\partial^2 W_n}{\partial T^2} \tag{20}$$

where $y = 1/x_1$ and P_n is replaced by F_n through (11). The finite difference form of (20) is obtained by replacing $\partial F_n/\partial y$ with the first difference approximation.

In the rectangle $0 \leq y \leq 1$, $-\pi/2 \leq x_3 \leq \pi/2$, (18) must be satisfied for $y = 1$. The boundary at infinity corresponds to $y = 0$ on which $F_n(0,x_3) = 0$. Assuming that $X_1(1,\pm\,\pi/2) = 0$, then since $P_n(y, \pm\,\pi/2)$ is bounded, it follows from (11) that $F_n(y, \pm\,\pi/2) = 0$. Given the normal shell displacements, W_n, and the incident acoustic pressure, P_{nI}, the boundary conditions together with (10), (11) and (13) determine the reflected and radiated acoustic pressure over the region external to the cavity defined by the surface of revolution.

Shell Equations

Solutions of the shell's equations of motion may be introduced into the shell fluid interaction problem either directly through finite difference approximation as in [2,3] or through a spectral representation as in [6]. Since there exists at present a more comprehensive library of shell programs which employ spectral representation rather than integration of a finite different approximation, the following development is based on spectral representation. For notational simplicity U_2, U_3, and W will note the coefficients corresponding to some given value of the summation index for the Fourier expansions, with respect to the polar angle $\bar{x}_2$, of the tangential, axial, and normal shell displacements respectively. Then in terms of generalized coordinates $q_n(T)$ and scaled normal modes U_{2m}, U_{3m} and W_m

$$W = \sum_{m=1}^{\infty} q_m(T)\, W_m \tag{21}$$

where

$$\frac{d^2 q_m}{dT^2} + \Omega_m^2 q_m = \frac{a^2}{c^2} G_m(T) \tag{22}$$

See [11]. $G_m(T)$ is given by

$$G_m(T) = -2\pi\rho_f c^2 a^3 \int P_T W_m X_1 \, dX_3 / N_m \tag{23}$$

where

$$N_m = 2\pi\rho_s h \, a^4 \int (U_{2m}^2 + U_{3m}^2 + W_m^2) X_1 \, dX_3 \tag{24}$$

ρ_s = shell density, h = shell thickness, $P_T = P + P_I + P_z$ where P_z is any pressure acting on the shell which is not applied through the fluid. For a given shell geometry the in vacuo scaled normal modes and scaled natural frequencies $\Omega_m = aW_m/c$ may be found by a variety of means, [11], and are therefore assumed to be known. The integral of the unknown surface acoustic pressure, P, in (23) may be approximated by

$$\int P \, W_m X_1 \, dX_3 \simeq \sum_k H_k \, P(k) \, W_m(k) \, X_1(k) \tag{25}$$

where $P(k) = P_{i,k}$, i corresponds to the row of nodes for which y = 1, k corresponds to the column number of nodes in the y, x_3 plane and H_k are the corresponding weights in any suitable numerical quadrature formula. Taking the Laplace transform of (22), assuming zero initial conditions, gives

$$q_m(s) = a^2 G_m(s) / (c^2 (s^2 + \Omega_m^2)) \tag{26}$$

Equations (10), (11), (14), (21), (25), (26) and boundary conditions are sufficient to determine the transform of the acoustic pressure $P_{i,k}$ over the region external to the shell and on the shell fluid interface together with the transform of the normal shell deflection. Solutions in the time domain are found through numerical inversion of the Laplace transform. See [12] and

[13].

Numerical Studies

In order to demonstrate the computational utility of the foregoing, numerical studies were carried out utilizing a shell of revolution, the upper right hand quadrant of which is shown in Fig. 2. The scaled dimensions of the shell are indicated on the figure together with crosses denoting the approximation realized by retaining five terms in the mapping function given by (3). The mid-section of the shell, $-1 \leq X_3 \leq +1$, is assumed to be an elastic cylindrical shell with an unscaled radius of 53.34 cm. (21 in), thickness of 2.54 cm (1 in), density of $7.829 \cdot 10^3$ kg/m^3 ($0.738 \cdot 10^{-3}$ lb $\cdot$ sec^2/in^4) with a length of 106.68 cm (42 in) for which $E = 2.0684 \cdot 10^{12}$ Pa ($30 \cdot 10^6$ psi) and $\nu = 0.3$. The cylindrical midsection is assumed to be pinned at the ends $X_3 = \pm 1$, $\bar{x}_3 = \pm 53.34$ cm ($\pm$ 21 in). The end sections of the shell, $|X_3| > 1$, are taken to be fixed rigid hemispherical caps. The shell is surrounded by an acoustic fluid for which $c = 1.4478 \cdot 10^3$ m/s ($0.575 \cdot 10^5$ in/sec) with a density of $\rho_f = 0.999 \cdot 10^3$ kg/m^3 ($0.935 \cdot 10^{-4}$ lb/sec^2/in^4). A suddenly applied scaled force, $P_z = \cos(\pi X_3/2)$, acts on the interior of the cylindrical midsection of the shell. Figure 3 shows the scaled normal displacement, as a function of scaled time, at the point $X_3 = 0$ for the shell surrounded by an acoustic medium and in vacuo. Figures 4 and 5 give results similar to those given in Figure 3 with $X_3 = 0.36726$ and 0.73451 respectively. Due to the symmetry of the loading, only normal modes W_m which are even functions of X_3 and the corresponding U_{3m} occur in (21) and (24). Of these, it was found necessary to sum only the first two in (21) and (24). The finite difference grid over the first quadrant of the mapped acoustic field consisted of 15 equally spaced rows and 8 unequally spaced columns of nodal points. The nodal column spacing in the mapped acoustic field is such that the corresponding spacing in the physical plane is uniform. The resulting system of equations was solved in double precision for the transforms of the shell displacements. Numerical inversions were carried out using [12].

Conclusions

A numerical solution for the transient vibration of an arbitrary shell of revolution, surrounded by an acoustic medium, has been formulated in terms of the conformally mapped acoustic equations, fluid shell boundary condition and spectral representation of the normal shell displacement. Numerical mapping techniques have been shown to be effective in this application. The technique readily includes the boundary condition at infinity and is exact within those approximations implicit in the finite difference method. The required in vacuo normal modes and natural frequencies may be computed utilizing anyone of the many dynamic shell analysis codes now available. Thus shell fluid interaction problems for shells with complex internal structure may be treated.

The present formulation, in terms of the Laplace transform, could be readily modified so that the time variable is suppressed through a finite difference technique. Steady state solutions may also be derived through a modification of the boundary condition at infinity.

Acknowledgements

The computer programming and numerical computations required for the development of the conformal mapping program were very capably performed by Mr. Milton Palmer. All computations were carried out at the University of Maryland Computer Science Center.

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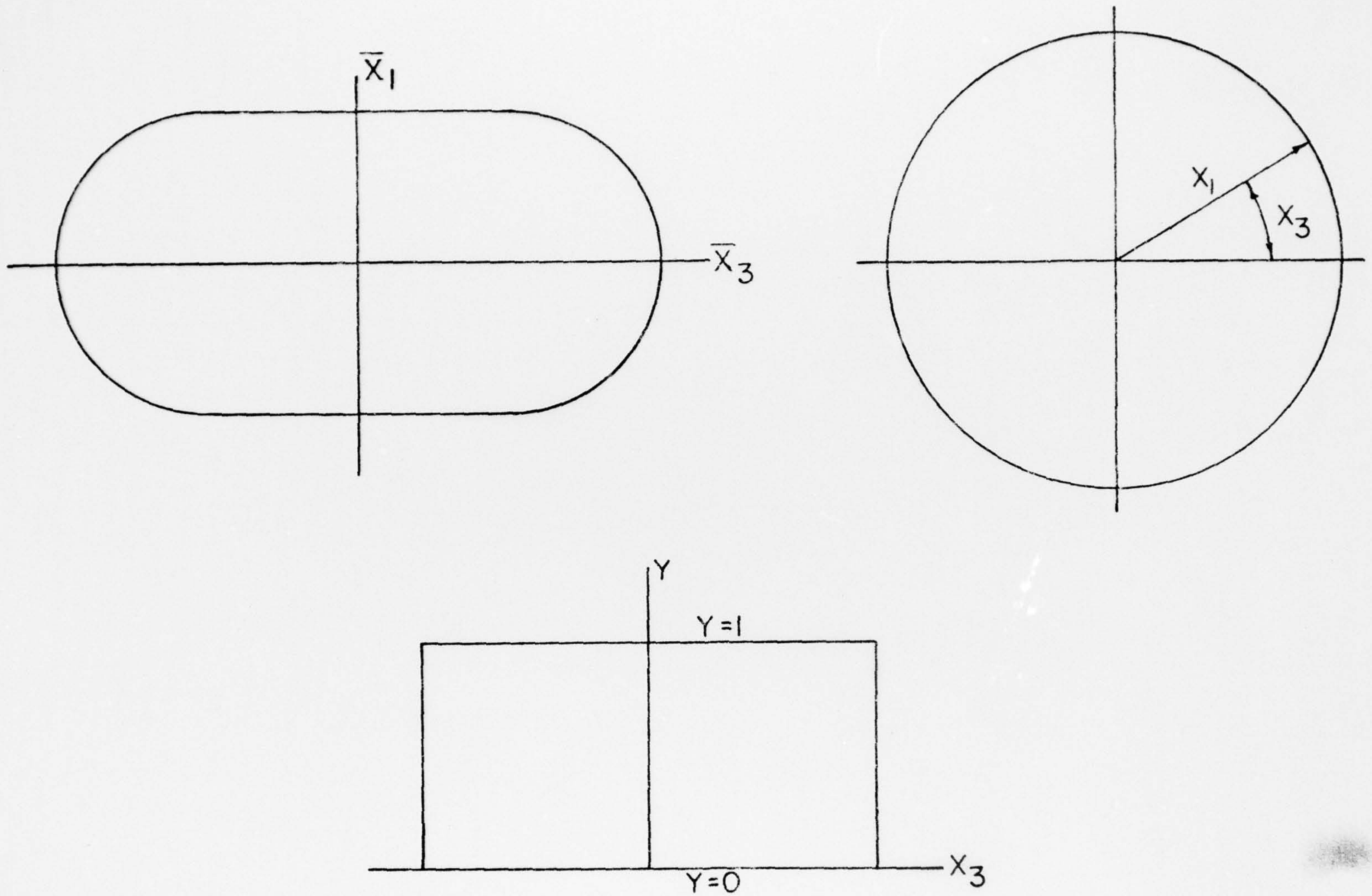
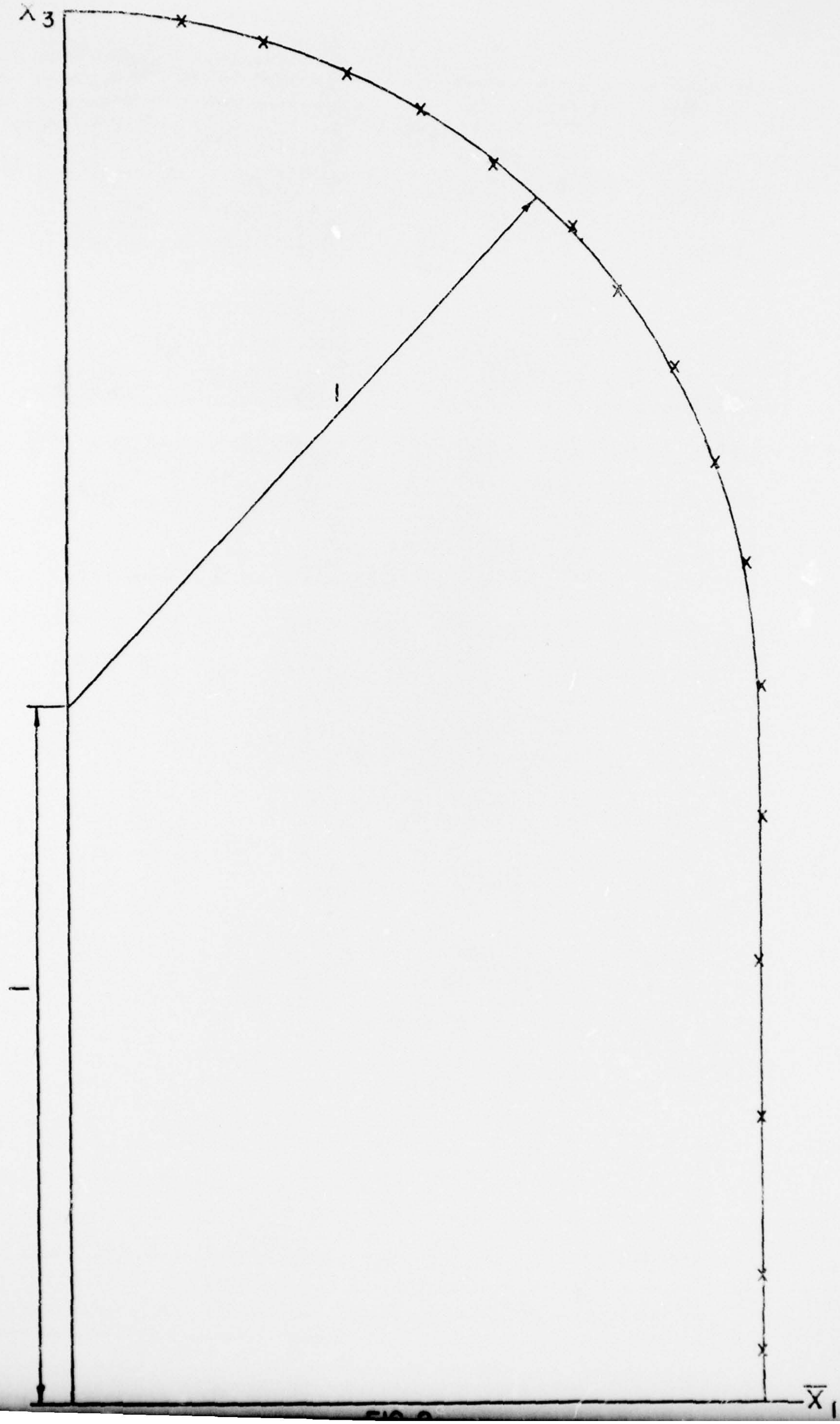


FIG. 1



X_3
1
1
$\bar{X}_1$

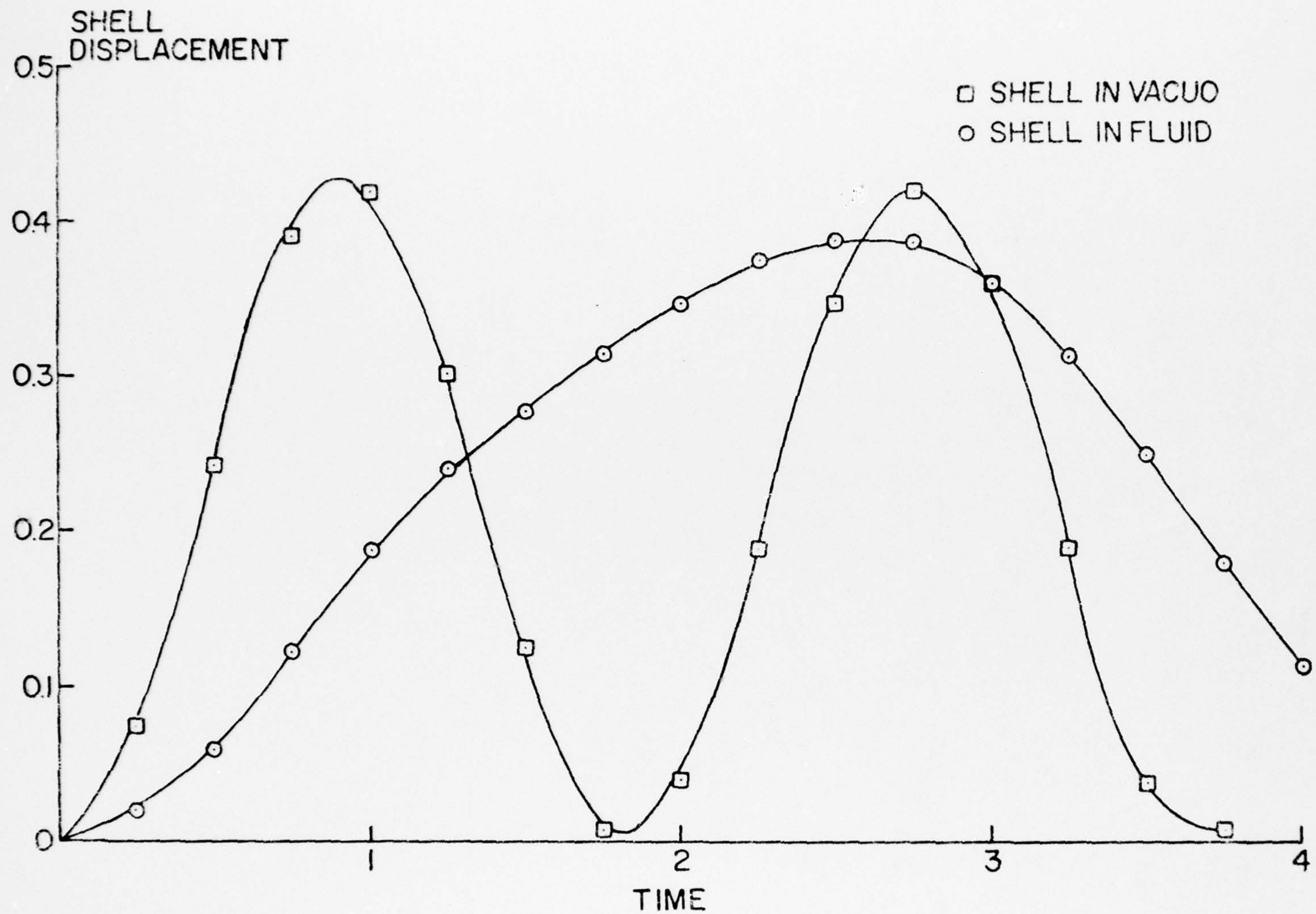

SHELL
DISPLACEMENT
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
TIME
SHELL IN VACUO
SHELL IN FLUID

FIG. 3

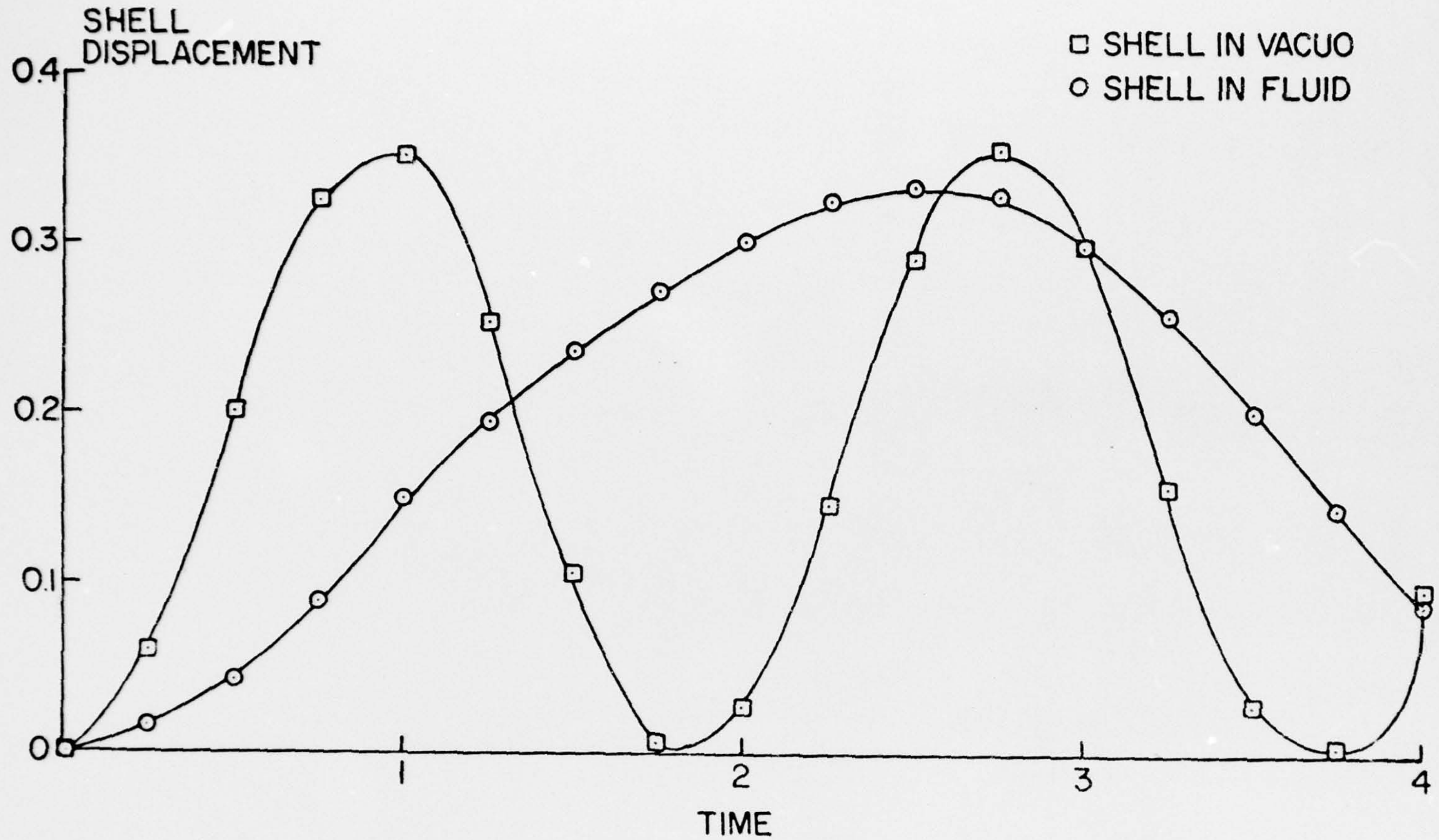

SHELL DISPLACEMENT
0.4
0.3
0.2
0.1
0
1
2
3
4
TIME
□ SHELL IN VACUO
○ SHELL IN FLUID

FIG. 4

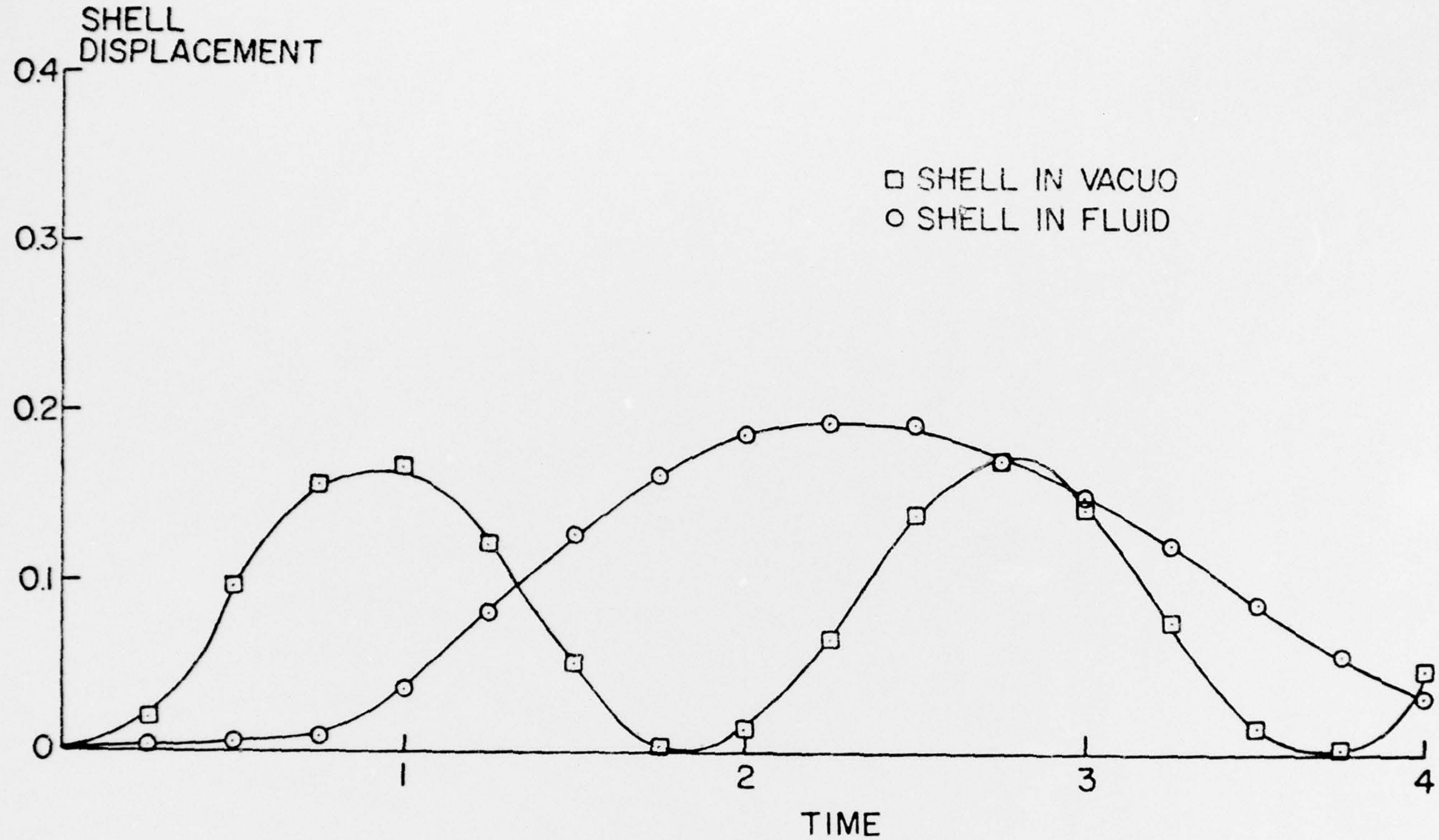

SHELL DISPLACEMENT
0.4
0.3
0.2
0.1
0
1
2
3
4
TIME
□ SHELL IN VACUO
○ SHELL IN FLUID

FIG. 5